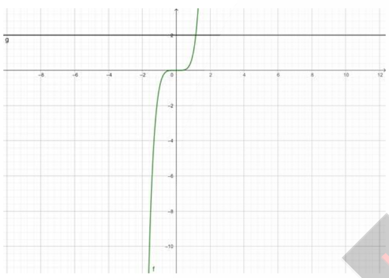
Solution

Class 12 - Mathematics

Class XII - 2020-2021 Paper -1

Part-A

1. Let x_1, $x_2 \in$ R be such that $f(x_1) = f(x_2)$
   $\Rightarrow x_1^3 = x_2^3$
   $\Rightarrow x_1 = x_2$
   Therefore, f is one-one function, hence $f(x) = x^3$ is injective.
2. x_1 ,x_2 are two different elements of R
   Let $f(x_1) = f(x_2)$
   $2x_1 = 2x_2$
   $x_1 = x_2$, hence f is one-one.
3. To show: f: R $\rightarrow$ R : : $f(x) = x^5$ is one - one and onto.
   Proof:
   $f(x) = x^5$
   $\Rightarrow y = x^5$

   Since the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) is one - one.
   The range of f(x) = ($-\infty$, ∞) = R(Codomain)
   $\therefore$ f(x) is onto
   Hence, showed f: R $\rightarrow$ R : $f(x) = x^5$ is one - one and onto.
4. Given sets are, A = {2, 3, 4}, B = {2, 5, 6, 7}
   Let g: A $\rightarrow$ B denote a mapping such that g = {(2,2),(3,5),(4,2)}
   We observe that $2, 4 \in A$ does not have unique image.
   Thus, g is not injective.
5. We have,
   $\sin^{-1}(\cos \frac{\pi}{9})$
   We know that $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$
   $= \sin^{-1}(\sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right))$
   $= \sin^{-1}(\sin\frac{7\pi}{18})$
   We know that $\sin^{-1}(\sin\theta) = \theta$
   $= \frac{7\pi}{18}$
   $\therefore \sin^{-1}(\cos\frac{\pi}{9}) = \frac{7\pi}{18}$
6. Let $y = \sin^{-1}\left(-\frac{1}{2}\right)$
   $\sin y = \frac{-1}{2}$
   $\sin y = \sin\left(\frac{-\pi}{6}\right)$
   Range of principal value of $\sin^{-1}$ is between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$
   Hence, the principal value is $\frac{-\pi}{6}$

7. $\begin{bmatrix} 2x-3y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\Rightarrow$ 2x – 3y = 1

$\Rightarrow$ x + y = 3

$\Rightarrow$ x = 3 – y

$\Rightarrow$ 2 (3 – y) – 3y = 1

$\Rightarrow$ -5y = -5

$\Rightarrow$ y = 1

$\Rightarrow$ x = 3 – 1

$\Rightarrow$ x = 2

8. According to the question,

$[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0 \Rightarrow [2x-9 \quad 4x]\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

$\Rightarrow [2x^2 - 9x + 12x] = [0]$

$\Rightarrow 2x^2 + 3x = 0$

$\Rightarrow$ x(2x + 3) = 0

$\therefore$ x = 0 or x = -3/2

9. According to the question, $A = [2 \quad 1 \quad 3]\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= [-2-1 \quad 1+3 \quad -2+3]\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = [-3 \quad 4 \quad 1]\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

= [-3 - 1] = $[-4]_{1\times 1}$

$\therefore$ Order of matrix A is 1×1.

10. According to the question, $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

Equating the corresponding elements,

$8 + y = 0$ and $2x + 1 = 5$

$\Rightarrow$ y = -8 and $x = \frac{5-1}{2} = 2$

$\therefore$ x - y = 2 - (-8) = 10

11. $2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

|2A| = 8 - 32 = -24

|A| = 2 - 8 = -6

4|A| = -24

Hence Proved.

12. Let $\Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$

Applying, $R_1 \to R_1 - 3R_2$ and $R_3 \to R_3 + 5R_2$ we get,

$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$, by expanding along the third row we get 12(0-0)=0.

So, Δ = 0

13. We can write $|2.AA'| = 4|A||A'|$
= 4×4×4 = 64

14. Here,
$$\begin{bmatrix}1 & 0 & 0\\ 0 & y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ -1\\ z\end{bmatrix} = \begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$ Since the LHS the first matrix is of order 3x3 and the second one of order 3x1
$$\Rightarrow \begin{bmatrix}x\\ -y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$
∴ x = 1, y = 0 and z = 1

15. LHS= $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$
$=\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) \left[\cot^{-1}x = \tan^{-1}\frac{1}{x}, x > 0\right]$
$= \tan^{-1}(a) - \tan^{-1}b + \tan^{-1}b - \tan^{-1}c + \tan^{-1}c - \tan^{-1}a = 0$

16. $\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) \left[\cot^{-1}x = \tan^{-1}\frac{1}{x},\ x > 0\right]$
= $\tan^{-1}(a) - \tan^{-1}b + \tan^{-1}b - \tan^{-1}c + \tan^{-1}c$ - tan-1a
= 0

Section I

17. i. (a) ₹ 100
ii. (c) ₹ 300
iii. (d) ₹ 1200
iv. (b) ₹ 1500
v. (c) ₹ 2700

18. Three equations are formed from the given statements:
3x + 2y + z = 2200
4x + y + 3z = 3100 and
x + y + z = 1200
Converting the system of equations in matrix form we get,
$$\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}2200\\ 3100\\ 1200\end{bmatrix}$$
i.e. AX = B
where A = $\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$, X = $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and B = $\begin{bmatrix}2200\\ 3100\\ 1200\end{bmatrix}$
⇒ $X = A^{-1}B$
$A^{-1} = \frac{1}{|A|}$ (Adj A)
|A| = 3(1 − 3) −2(4 − 3) + 1(4 − 1) = −6 −2 + 3 = −5
Adj A = $\begin{bmatrix}-2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5\end{bmatrix}$
∴ $A^{-1} = \frac{1}{-5}\begin{bmatrix}-2 & -1 & 5\\ -1 & 2 & -5\\ 3 & -1 & -5\end{bmatrix}$
$= \frac{1}{5}\begin{bmatrix}2 & 1 & -5\\ 1 & -2 & 5\\ -3 & 1 & 5\end{bmatrix}$

$\Rightarrow X = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 4400+3100-6000 \\ 2200-6200+6000 \\ -6600+3100+6000 \end{bmatrix}$

$= \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$

$\Rightarrow$ x = 300, y = 400 and z = 500

i.e. The award money for each value are Rs.300 for Tolerance, Rs.400 for Kindness and Rs.500 for Leadership.

i. (b) 300
ii. (d) 500
iii. (b) 400
iv. (c) A is a zero matrix
v. (a) B

Section II

19. We have to show that,
$\tan^{-1}\left(\frac{2}{3}\right) = \frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$
LHS = $\tan^{-1}\left(\frac{2}{3}\right)$
Dividing and multiplying by 2,
$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{2}{3}\right)\right\}$
$= \frac{1}{2}\left\{\tan^{-1}\left(\frac{2\left(\frac{2}{3}\right)}{1-\left(\frac{2}{3}\right)^2}\right)\right\}$ {Since $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$}
$= \frac{1}{2}\tan^{-1}\left(\frac{\frac{4}{3}}{\frac{5}{9}}\right)$
$= \frac{1}{2}\tan^{-1}\left(\frac{4}{3} \times \frac{9}{5}\right)$
$= \frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$
$\tan^{-1}\left(\frac{2}{3}\right) = \frac{1}{2}\tan^{-1}\left(\frac{12}{5}\right)$

20. Let $\cos^{-1}x = \theta$, then $\cos\theta = x$, where $\theta \in [0, \pi]$
Therefore, $\tan\left(\cos^{-1}x\right) = \tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta} = \frac{\sqrt{1-x^2}}{x}$
Hence, $\tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{\sqrt{1-\left(\frac{8}{17}\right)^2}}{\frac{8}{17}} = \frac{15}{8}$

21. f : Z $\rightarrow$ Z given by f(x) = x^2
Since, $z = \{0, \pm1, \pm2, \pm3, \ldots\}$ therefore, f(-1) = f(1) = 1
$\Rightarrow$ -1 and 1 have same image. $\therefore$ f is not injective.
There are such numbers of co-domain which have no image in domain Z.
e.g. 3 $\in$ co-domain, but $\sqrt{3} \notin$ domain of f. $\therefore$ f is not surjective.

OR

Case I : When x > 0.Then,we have,
|x| = x
$\Rightarrow \frac{1}{\sqrt{|x|-x}} = \frac{1}{\sqrt{x-x}} = \frac{1}{0} = \infty$
Case II : When x < 0
|x| = -x
$\Rightarrow \frac{1}{\sqrt{|x|-x}} = \frac{1}{\sqrt{-x-x}} = \frac{1}{\sqrt{-2x}}$ (exists because when x < 0, -2x > 0)
$\Rightarrow$ f(x) is defined when x < 0
Therefore, domain = $(-\infty, 0)$

22. A = $\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$

$(A - B)^T = A^T - B^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

L.H.S = R.H.S.

23. Let $\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

$[R_2 \to R_2 - R_1 \ and \ R_3 \to R_3 - R_1]$

$$= \begin{vmatrix} 1 & x & y \\ 0 & x+y-y & 0 \\ 0 & 0 & x+y-y \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$$

Expanding along Ist column

$$= 1\begin{vmatrix} y & 0 \\ 0 & x \end{vmatrix}$$

= xy

OR

Area of triangle=35 units

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{bmatrix} = \pm 35$$

Expanding along row Ist,

$\Rightarrow \frac{1}{2}$[2(4 - 4)+ 6(5 - k)+1(20 - 4k)] = ± 35

$\Rightarrow \frac{1}{2}$[30 - 6k + 20 - 4k] = ± 35

$\Rightarrow \frac{1}{2}$[50 - 10k] =± 35

$\Rightarrow$25 - 5k = ± 35

$\Rightarrow$25 - 5k = 35 or 25 - 5k = -35

$\Rightarrow$-5k = 10 or 5k = 60

$\Rightarrow$k = -2 or k = 12

24. 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

Here, $a_{ij} = \frac{1}{2}|-3i + j|$

$\therefore \ a_{11} = \frac{1}{2}|-3 \times 1 + 1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$

$a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$

$a_{31} = \frac{1}{2}|-3 \times 3 + 1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$

$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$

$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$

$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}$

$a_{13} = \frac{1}{2}|-3\times 1+3| = \frac{1}{2}|-3+3| = 0$
$a_{23} = \frac{1}{2}|-3\times 2+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$
$a_{33} = \frac{1}{2}|-3\times 3+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$
$a_{14} = \frac{1}{2}|-3\times 1+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$
$a_{24} = \frac{1}{2}|-3\times 2+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$
$a_{34} = \frac{1}{2}|-3\times 3+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

25. We have to compute ,

5A - 3B + 4C $= 5\left(\begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix}\right)$ - 3 $\left(\begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix}\right) + 4\left(\begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}\right)$

$= \left(\begin{bmatrix} 0 & 5 & -10 \\ 25 & -5 & -20 \end{bmatrix}\right) - \left(\begin{bmatrix} 3 & -9 & -3 \\ 0 & -6 & 15 \end{bmatrix}\right) + \left(\begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}\right)$

$= \begin{bmatrix} -3 & 14 & -7 \\ 25 & 1 & -35 \end{bmatrix} + \begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}$

$= \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix}$

26. Let P (x, y) be any point on AB. Then the equation of line AB is,

$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$

y = 3x

Area $\Delta ABD = 3$ square unit

$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ K & 0 & 1 \end{vmatrix} = \pm 3$

$k = \pm 2$

27. To prove: function is neither one-one nor onto function.

Given: f : R → R : f (x) = x^4

We have,

f(x) = x^4

For, $f(x_1) = f(x_2)$

= $x_1^4 = x_2^4$

= $(x_1^4 - x_2^4) = 0$

=$(x_1^2 - x_2^2)(x_1^2 + x_2^2) = 0$

= $(x_1 - x_2)(x_1 + x_2)(x_1^2 + x_2^2) = 0$

= $x_1 = x_2$ or, $x_1 = -x_2$ or, $x_1^2 = -x_2^2$

We are getting more than one value of x_1 (no unique image)

∴ f(x) is not one-one function.

f(x) = x^4

Let f(x) = y such that $y \in R$

= y = x^4

$\Rightarrow x = \sqrt[4]{y}$

If y = -2, as $y \in R$

Then x will be undefined as we can't place the negative value under the square root

Hence f(x) is not onto function.
Hence Proved.

OR

We have,

$$f(n)=\begin{cases} n+1 & \text{if} \quad n \text{ is odd} \\ n-1 & \text{if} \quad n \text{ is even} \end{cases}$$

Injectivity :
Case I: If n is odd,
Let x, y $\in$ N such that f (x)=f (y)
As, f (x) = f (y)
$\Rightarrow$ x + 1 = y + 1
$\Rightarrow$ x = y
Case II: If n is even,
Let x, y $\in$ N such that f (x)=f (y)
As, f (x) = f (y)
$\Rightarrow$ x - 1 = y - 1
$\Rightarrow$ x = y
Therefore, f is injective.
Surjectivity:
Case I: If n is odd,
As, for every n $\in$ N, there exists y = n - 1 in N such that
f (y) = f(n - 1) = n - 1 + 1 = n
Case II: If n is even,
As, for every n $\in$ N, there exists y = n + 1 in N such that f (y) = f (n + 1) = n + 1 - 1 = n
Therefore, f is surjective.
Hence, f is a bijection

28. We know that: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
Put $\cos\theta = x$
$\Rightarrow \theta = \cos^{-1}x$
$\therefore \cos 3\theta = 4x^3 - 3x$
$\Rightarrow 3\theta = \cos^{-1}\left(3x - 4x^3\right)$
Putting $\theta = \cos^{-1}x$,
$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$
Hence Proved.

Part-B

29. Let the charges for each page of English and Hindi be Rs x and Rs y, respectively.
Then, according to the given condition, we have
10x + 3y = 145
and 3x + 10y = 180
Above system of equations can be written in matrix form as
AX = B.........(i)
where, A = $\begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 145 \\ 180 \end{bmatrix}$
Now, |A| = $\begin{vmatrix} 10 & 3 \\ 3 & 10 \end{vmatrix}$ = 100 - 9 = 91
$\Rightarrow |A| \neq 0$, hence unique solution exists.
Now, co-factors of the elements of |A| are
$C_{11} = (-1)^2\ 10 = 10$
$C_{12} = (-1)^2 3 = -3$
$C_{21} = (-1)^3 3 = -3$
and $C_{22} = (-1)^4\ 10 = 10$

$\therefore \quad \text{adj}(A) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$= \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}^T = \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{91}\begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}$

From Eq(i), we get

$\Rightarrow X = A^{-1}B$

$\Rightarrow \quad X = \frac{1}{91}\begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}\begin{bmatrix} 145 \\ 180 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{91}\begin{bmatrix} 1450 - 540 \\ -435 + 1800 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{91}\begin{bmatrix} 910 \\ 1365 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$

On equating the corresponding elements, we get x = 10 and y = 15

Hence, charges for each English page is Rs 10 and each Hindi page is Rs 15.

Since, the typist charge only Rs 2 per Hindi page from a poor student Shyam.

$\therefore$ Cost of typing 5 Hindi pages for Shyam = 2×5 = Rs 10.

But the original amount of typing 5 Hindi pages = 5×15 = Rs 75

Hence, the typist charge Rs 65(Rs 75- 10) less amount from poor boy.

Care and concern towards weaker section of the society.

30. L.H.S. $= \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Taking common a,b,c from R_1, R_2, R_3 respectively,

$= abc\begin{vmatrix} -a & b & c \\ a & -b & b \\ a & b & -c \end{vmatrix}$

[operating $R_1 \to R_1 + R_2$]

$= abc\begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ b & b & -c \end{vmatrix}$

Expanding along C_3

$= abc.2c\begin{vmatrix} a & -b \\ a & b \end{vmatrix}$

= abc.2c(ab + ab)

= abc.2c.2ab = $4a^2b^2c^2$ = R.H.S

31. A^2 = A.A

$= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

A^2 = kA - 2I.

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$

$\Rightarrow$ 3k = 3

$\Rightarrow$ k = 1

OR

Let P(n) be $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

For n = 1

$A^1 = \begin{bmatrix} \cos 1.\theta & \sin 1.\theta \\ -\sin 1.\theta & \cos 1.\theta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ = A

P(n) is true for n = 1

Let us assume that P(n) is true for n = k

$A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

For n = k+ 1

$A^{k+1} = A.A^k$

$= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} . \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$

$= \begin{bmatrix} \cos\theta\cos k\theta - \sin\theta\sin k\theta & \cos\theta\sin k\theta + \sin\theta\cos k\theta \\ -\sin k\theta\cos k\theta - \cos\theta\sin k\theta & -\sin\theta\sin k\theta + \cos\theta\cos k\theta \end{bmatrix}$

$= \begin{bmatrix} \cos(\theta + k\theta) & \sin(\theta + k\theta) \\ -\sin(\theta + k\theta) & \cos(\theta + k\theta) \end{bmatrix}$

$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$

Thus, P(n) is true for n = k + 1.

Hence P(n) is true for every n in N.

32. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$ = 1(10 - 0) - 2(0 - 0) + 3(0 - 0) $= 10 \neq 0$

$\therefore A^{-1}$ exists.

$A_{11} = +\begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix} = +(10-0) = 10, A_{12} = -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -(0-0) = 0$

$A_{13} = +\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = +(0-0) = 0, A_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -(10-0) = -10$

$A_{22} = +\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = +(5-0) = 5, A_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0$

$A_{31} = +\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = +(8-6) = 2, A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4-0) = -4$

$A_{33} = +\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = +(2-0) = 2$

$\therefore adj.A = \begin{vmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{vmatrix}'$

$$= \begin{vmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{vmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj.A = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

OR

Matrix form of given equations is AX = B

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

= 1(12 - 5) - (-1)(9 + 10) + 2(-3 - 8)

= 7 + 19 - 22 = 4 $\neq$ 0

A_{11} = 7, A_{12} = -19, A_{13} = -11

A_{21} =1, A_{22} = -1, A_{23} = -1

A_{31} = -3, A_{32} =11, A_{33} = 7

adj A= $\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(adj.A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Therefore, x = 2, y = 1 and z = 3

33. When n = 1

$(aI+bA)^1=a^1I + 1.a^{1-1}\,bA$

aI + bA = aI + bA

L.H.S = R.H.S

The result is true for n = 1.

When n = k

$(aI + bA)^K = a^KI + Ka^{K-1}bA$..........; (i)

Assume that the result is true for n = k

When n = k + 1

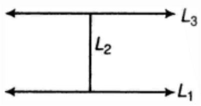
$(aI + bA)^{k+1} = (aI + bA). (aI + bA)^k$

$= (aI + bA). (a^kI + ka^{k-1}bA)$ [From (i)]

$= a^{k+1}I + ka^kbA + a^kbA + ka^{k-1}\, b^2A^2 \begin{bmatrix} \because II = I \\ IA = A = AI \end{bmatrix}$

= a^{k+1} I+ (k+1) a^kbA $\left[\because A^2=0\right]$
Hence result is true for n = k+1, when ever it is true for n = k
Hence ,by the principle of mathematical induction the result is true for all n in N.

34. For n = 1,we have, $AB^1 = B^1A$
$\Rightarrow$ AB = BA, which is true.
Let it be true for n = m i.e $AB^m = BA^m$.........(1)
Then,for n = m + 1,
$AB^{m+1} = A(B^m B) = (AB^m)B = (B^mA)B$ [by (1)]
$= B^m(AB) = (B^mB)A = B^{m+1}A$.
So ,it is true for n=m+1
$\therefore AB^n = B^nA$

35. Given, R = $\{(L_1,L_2): L_1 \perp L_2\}$
and A = Set of all lines in a plane
Reflexive Let L $\in$ A and (L, L) $\in$ R $\Rightarrow$ L $\perp$ L
But any line cannot be perpendicular to itself, which is not true.
$\therefore$(L, L) $\notin$ R $\forall$ L $\in$ A
So, R is not reflexive.
Symmetric Let L_1 , L_2 $\in$ A and (L_1 , L_2) $\in$ R
$\Rightarrow L_1 \perp L_2$ (two lines are perpendicular to each other)
$\Rightarrow L_2 \perp L_1$
$\Rightarrow (L_2, L_1) \in R, \forall (L_1, L_2) \in R$
So, R is symmetric.
Transitive Let L_1 , L_2 and L_3 $\in$ A
then (L_1 , L_2) $\in$ R $\Rightarrow L_1 \perp L_2$
$(L_2, L_3) \in R \Rightarrow L_2 \perp L_3$
But L_1 is not perpendicular to L_3 as shown in figure


Then $(L_1, L_3) \notin R$
Thus, $(L_1, L_2) \in R$, $(L_2, L_3) \in R$
$\Rightarrow (L_1, L_3) \in R$
So, R is not transitive.
In our real life relation between two friends is a symmetric relation.

Section III

36. Given: Matrix $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{vmatrix}$

$\Rightarrow$ |A| = 2(-4 + 4) - (-3)(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 $\neq$ 0
$\therefore$ A^{-1} exists and $A^{-1}=\frac{1}{|A|}(adj.A)$...(i)
Now, A_{11} = 0, A_{12} = 2, A_{13} = 1 and A_{21} = -1, A_{22} = -9, A_{23} = -5 and A_{31} = 2, A_{32} = 23, A_{33} = 13

$\therefore adj.A=\begin{bmatrix}0 & 2 & 1\\ -1 & -9 & -5\\ 2 & 23 & 13\end{bmatrix}'=\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$

From eq. (i),

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, Matrix form of given equations is AX = B

$$\Rightarrow \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Here $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Therefore, solution is unique and X = $A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z = 3

OR

Given the purchase details of three shopkeeper A, B, and C.

A: 12 dozen notebooks, 5 dozen pens, and 6 dozen pencils

B: 10 dozen notebooks, 6 dozen pens, and 7 dozen pencils

C: 11 dozen notebooks, 13 dozen pens, and 8 dozen pencils

Hence, the items purchased by A, B, and C can be represented in matrix X of order 3x3 where the rows denoting the each shopkeeper and columns denoting the number of dozens of items as –

$$X = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix}$$

The price of each of the items is also given.

Cost of one notebook = 40 paise

$\Rightarrow$ Cost of one dozen notebooks = 12 $\times$ 40 paise

$\Rightarrow$ Cost of one dozen notebooks = 480 paise

$\therefore$ Cost of one dozen notebooks = ₹ 4.80

Cost of one pen = ₹ 1.25

$\Rightarrow$ Cost of one dozen pens = 12 $\times$ ₹ 1.25

$\therefore$ Cost of one dozen pens = ₹ 15

Cost of one pencil = 35 paise

$\Rightarrow$ Cost of one dozen notebooks = 12 $\times$ 35 paise

$\Rightarrow$ Cost of one dozen notebooks = 420 paise

$\therefore$ Cost of one dozen notebooks = ₹ 4.20

Hence, the cost of purchasing one dozen of the items can be represented in matrix form with each row corresponding to an item as –

$$Y = \begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$$

Now, the individual bill for each shopkeeper can be found by taking the product of the matrices X and Y.

The product is feasible because the order of X is 3x3 and that of Y is 3x1 ,which makes the number of columns of x is equal to the number of the columns of Y.The product is given as follows.

$$XY = \begin{bmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{bmatrix}\begin{bmatrix} 4.80 \\ 15 \\ 4.20 \end{bmatrix}$$

$$\Rightarrow XY = \begin{bmatrix} 12 \times 4.80 + 5 \times 15 + 6 \times 4.20 \\ 10 \times 4.80 + 6 \times 15 + 7 \times 4.20 \\ 11 \times 4.80 + 13 \times 15 + 8 \times 4.20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 57.60 + 75 + 25.20 \\ 48 + 90 + 29.40 \\ 52.80 + 195 + 33.60 \end{bmatrix}$$

$$\therefore XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

Thus, the bills of shopkeepers A, B and C are ₹ 157.80, ₹ 167.40 and ₹ 281.40 respectively.

37. $B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

Let $P = \frac{1}{2}(B + B') = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$

$$P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$$

Thus $P = \frac{1}{2}(B + B')$ is a symmetric matrix

Let $Q = \frac{1}{2}(B - B') = \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$

$$Q' = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{2} \\ -\frac{1}{2} & 0 & -3 \\ -\frac{5}{2} & 3 & 0 \end{bmatrix}$$

$$Q' = -\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

Q' = - Q

Thus $Q = \frac{1}{2}(B - B')$ is a skew symmetric matrix

$$P + Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

= A

Hence proved.

OR

R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty
For any a∈A, |a−a|=0 Which is divisible by 2.
∴ (a, a) ∈ r for all a ∈ A
So ,R is Reflexive
Symmetric :
Let $(a, b) \in R$ for all a, b ∈ R
|a−b| is divisible by 2
|b−a| is divisible by 2
$(a, b) \in r \Rightarrow (b, a) \in R$
So, R is symmetirc .
Transitive :
Let $(a, b) \in R$ and $(b, c) \in R$ then
$(a, b) \in R$ and $(b, c) \in R$
|a−b| is divisible by 2
|b−c| is divisible by 2
Two cases :
Case 1:
When b is even
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
∴ (a, c) ∈ R
Case 2:
When b is odd
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
So R is transitive.
Hence , R is an equivalence relation

38. Given, $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = a$

$\left[\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)\right]$

$\Rightarrow \cos^{-1}\left[\frac{x}{a}.\frac{y}{b} - \sqrt{1-\frac{x^2}{a^2}}.\sqrt{1-\frac{y^2}{b^2}}\right] = a$

$\Rightarrow \frac{xy}{ab} - \sqrt{1-\frac{x^2}{a^2}} \cdot \sqrt{1-\frac{y^2}{b^2}} = \cos a$

$\Rightarrow \frac{xy}{ab} - \cos a = \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$

On squaring both side, we get

$\Rightarrow \left(\frac{xy}{ab} - \cos a\right)^2 = \left(\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)^2$

$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2 a - 2.\frac{xy}{ab}.\cos a = \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$

$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2 a - 2\frac{xy}{ab}\cos a = 1 - \frac{y^2}{b^2} - \frac{x^2}{a^2} + \frac{x^2y^2}{a^2b^2}$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos a = 1 - \cos^2 a$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos a = \sin^2 a$

OR

We have , |A| = $\begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

=2[-4+4]+3[-6+4]+5[3-2]=0+3(-2)+5(1)=-6+5=-1$\neq 0$

$\therefore$ A^{-1} exists

Now, A_{11} = 0, A_{12} = 2, A_{13} = 1

A_{21} = -1, A_{22} = -9, A_{23} = -5

A_{31} = 2, A_{32} = 23, A_{33} = 13

$$\therefore adjA = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA)$$

$$= \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equation can be written is Ax = B, X = A^{-1}B

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$ replace column matrix of RHS by $\begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 5 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x = 1

y = 2

z = 3